Research on Constructing a Teaching Pathway for Primary School Mathematics Spatial Awareness Oriented to Performance Goals — A Case Study of "Observing Objects"

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Abstract: The Compulsory Education Mathematics Curriculum Standards (2022 Edition) regards spatial awareness as one major manifestation of core competencies. Nevertheless, frontline teachers lack clear operational pathways to stratify and implement spatial awareness in specific teaching content. Taking "Observing Objects" as the teaching carrier, this paper adopts performance goals as a tool to translate core competencies into observable behaviors, and develops a four-tier performance goal framework for spatial awareness: identification, representation, abstraction and generalization. Based on this framework, a three-stage progressive teaching pathway is designed: establishing the correspondence between directions and views via hands-on operation, realizing the conversion between three-dimensional solids and views through imaginal operation, and forming structured approaches to spatial reasoning by means of variant transfer. This study argues that the framework of performance goals offers a transformation mechanism for spatial awareness instruction to shift from experience-based description to goal-oriented guidance. As a theoretical construction research, the proposed teaching pathway can provide references for the instructional design of "Observing Objects".
Key words: spatial awareness; performance goals; instructional pathway

Introduction

Spatial awareness is one of the core competencies explicitly identified in the Mathematics Curriculum Standards for Compulsory Education (2022 Edition), and "Observing Objects" is a typical carrier for cultivating this competency. In teaching practice, however, although students can observe objects and describe views under the guidance of teachers, once the objects are removed, they are often at a loss when facing a new solid figure or a reverse reasoning task. This phenomenon reflects the key crux of teaching: teaching objectives remain at general statements such as "cultivating a sense of space", lacking a specific depiction of the levels of students' cognitive development, so that teaching behavior lacks precise direction. Performance goals, as a way of stating objectives that translates core competencies into observable and evaluable behavioral performances of students in specific tasks, provide a transformation tool for teaching to move from "experience description" to "goal orientation". In view of this, this paper takes "Observing Objects" as the carrier, constructs a hierarchical framework of performance goals for spatial awareness, and on this basis discusses a progressive teaching pathway.

1. Hierarchical Construction of Performance Goals for Spatial Awareness

1.1 Cognitive Development of Spatial Awareness and the Teaching Positioning of "Observing Objects"

The Mathematics Curriculum Standards (2022 Edition) define spatial awareness as "the intuition and understanding of the shape, size and positional relations of spatial objects or figures", whose core is students' dynamic ability to convert, operate and reason between "objects" and "figures". From the perspective of cognitive development, primary school students' spatial thinking follows a progressive path of "intuitive perception → operational verification → imaginal operation → abstract reasoning". This process implies that students must first accumulate spatial experience through object operation, then gradually internalize it into mental images, and finally be able to reason about and judge spatial relations with the objects removed. "Observing Objects" is exactly typical content that matches this cognitive path: it requires students to observe solid figures from different directions, to identify and draw views, and then to infer the solid structure reversely from the views. It involves the screening and projection of information from three dimensions to two dimensions, as well as the synthesis and reconstruction of information from two dimensions to three dimensions, thereby providing students with a development channel from "operational perception" to "spatial reasoning".

1.2 Performance Goals: A Transformation Tool Moving Spatial Awareness from "Implicit" to "Observable"

The particularity of spatial awareness lies in its implicitness: it occurs in the individual's mind, cannot be directly assessed, and can only be inferred from students' explicit behaviors in specific tasks. Can they accurately describe a view? Can they complete an assembly after the object has been removed? Can they explain the reasoning process? These "what one can do" are precisely the concrete embodiment of spatial awareness. In actual teaching, however, teachers are accustomed to describing objectives with general statements such as "cultivating spatial imagination", which leaves teaching without precise direction. Performance goals emphasize observable and evaluable learning behaviors of students in specific situations, and point to "what one can do" rather than "what one knows". Their core significance lies in providing a technical path of "goal stratification": through stratified descriptions of the behavioral characteristics of students at different cognitive stages, teachers can "see" to which step students' thinking has developed and where to go next.

1.3 Performance Goal Levels of Spatial Awareness in "Observing Objects"

Van Hiele's theory of levels of geometric thinking points out that students' geometric thinking develops level by level from visual recognition, property analysis and logical reasoning to formal deduction. This basic logic of "from overall perception to property analysis and then to relational reasoning" provides an important reference for the stratification of spatial awareness. On this basis, combining the laws of cognitive development of spatial awareness with the characteristics of the learning tasks of "Observing Objects", this paper proposes that the performance goals of spatial awareness in "Observing Objects" can be divided into four levels: identification, representation, abstraction and generalization (see Table 1).

Table 1 Performance goal levels of spatial awareness in "Observing Objects"

Level	Element	Specific description
Level 1	Identification	Can perceive and identify the shapes seen when observing a solid figure from the front, top and side, establish a one-to-one correspondence between "observation direction" and "view shape", and complete direction–view matching. Cognitive characteristics: at the level of spatial perception, relying on direct perception of objects or intuitive diagrams.
Level 2	Representation	Can accurately express the observed views by means of language description, sketch drawing or physical arrangement. Cognitive characteristics: a stable visual image has been formed and can be externalized through external media.
Level 3	Abstraction	Can, without the object, infer the possible structure of the solid figure in the mind according to the views, complete the reverse conversion between views and the solid, and realize that the same view may correspond to several assembly methods. Cognitive characteristics: a stable mental image has been formed, spatial relations can be operated in the mind, and external objects are no longer relied upon.
Level 4	Generalization	Can identify the common structure of spatial relation problems in different situations, use transferable strategies to solve variant problems, and generalize the general method of "direction decomposition → item-by-item extraction → comprehensive restoration". Cognitive characteristics: a transferable spatial reasoning structure has been formed, and the reasoning process can be reflected upon and adjusted.

The four levels provide a reference framework for teaching: Levels 1 and 2 constitute the foundation layer; Level 3 is the core development layer, representing the key transition from reliance on external operation to internal imaginal operation; Level 4 is the extension layer, pointing to the deepening and transfer of spatial reasoning. The functions of this framework are embodied in three aspects: a concrete description of learning objectives, a basis for the design of teaching pathways, and a reference standard for teaching evaluation.

2. Construction of the Teaching Pathway: Instructional Design Around Performance Goals

The analytical framework above divides the development of spatial awareness in "Observing Objects" into four levels: identification, representation, abstraction and generalization, providing clear goal orientation for teaching. Objectives, however, do not automatically turn into students' ability development; a teaching pathway matching them has to be designed. The pathway proposed in this paper follows a core logic: "perceiving direction in operation → establishing correspondence in representation → realizing conversion in reasoning → forming structure in transfer". The three stages undertake different teaching functions: Stage 1 completes the establishment of a "sense of direction" and the preliminary expression of "views"; Stage 2 completes the shedding of "object dependence" and the operational training of mental images; Stage 3 completes the abstraction of concrete experience and the generalization of reasoning strategies. The instructional design corresponding to each level is elaborated stage by stage below.

2.1 Stage 1: Establishing the "Direction–View" Correspondence in Object Operation

Level 1 (identification) and Level 2 (representation) are situated at the level of spatial perception and image externalization, where students need object operation as a cognitive scaffold to establish the correspondence between "direction names" and "view shapes". The teaching positioning of this stage is to complete the transition from "intuitive observation" to "purposeful directional observation", which is divided into three progressive links.

The first link is free observation and preliminary expression: the teacher presents a solid figure assembled from several small cubes, lets students observe it freely and say "what do you see", thereby exposing their initial observation habits. The second link is precise matching and the establishment of correspondence: the teacher provides view cards for three standard directions, and students complete "direction–view" matching through actual observation; the teacher emphasizes the linguistic correspondence between "line-of-sight direction" and "view shape", and guides students who match incorrectly back to the object to correct themselves. The third link is multiple representation and preliminary externalization: students choose one direction and express the shape they see by drawing, verbal description or gestures, and the teacher displays different forms of expression and guides the whole class to identify them.

2.2 Stage 2: Realizing the "Solid–View" Conversion in Imaginal Operation

Level 3 is the key turning point in the development of spatial awareness: students need to move from "drawing views while looking at the object" to "completing the mutual conversion between the solid and views in the mind without looking at the object". The core of this leap is the formation of "imaginal operation" ability, namely the ability of students to operate mental images of figures in the mind instead of relying on external objects.

Students usually face three cognitive difficulties in making this leap: the "information compression" from three dimensions to two dimensions, namely uncertainty about which details can be ignored when looking from a certain direction; the "information restoration" from two dimensions to three dimensions, namely difficulty in inferring the solid structure corresponding to a view; and the inertia of a fixed perspective, namely thinking only from the orientation one is accustomed to.

Two progressive tasks are set at this stage. Task 1 (views → solid): the teacher provides three views, and students assemble possible solid figures with small cubes, experiencing in operation that "the same view can correspond to multiple solid structures". Task 2 (solid → views): a solid figure is presented quickly and then covered, and students draw the views from memory, forcing a shift from "real-time perception" to "imaginal operation". The key node of teacher intervention is when students experience cognitive conflict: by asking "how did you count them" and "how are those faces arranged", the teacher guides students to externalize the reasoning process and reach consensus through discussion.

2.3 Stage 3: Forming Structured Methods of Spatial Reasoning in Variant Transfer

Level 4 points to the ability to "solve a class of problems", that is, students can consciously invoke the thinking strategy of "direction decomposition → item-by-item extraction → comprehensive restoration" when facing a new solid figure. The core task of this stage is to design a set of progressively graded variant exercises: maintaining the structural commonality of "conversion between the solid and views" while changing the complexity of the figure, the completeness of information and the openness of the answer. Taking "assembling a solid figure according to views" as an example, the three progressive levels are: complete three views (unique structure) → one view missing (multiple assembly methods) → only one view given (enumerate all possibilities and explain how to ensure that nothing is omitted). After completing each level, students are guided to compare similarities and differences, and finally to summarize the general method in their own words.

In summary, the three stages constitute a teaching pathway that advances layer by layer: Stage 1 moves spatial perception from "blurred" to "clear"; Stage 2 moves spatial thinking from "reliance on the external" to "internal operation"; Stage 3 moves spatial reasoning from "concrete situations" to "general methods". This pathway corresponds precisely to the four performance goal levels in Table 1, providing teachers with a clear action framework for implementing the teaching of spatial awareness.

3. Implementation Points of the Teaching Pathway and Responses to Key Problems

The preceding section constructed a teaching pathway for spatial awareness oriented to performance goals. This chapter

discusses the issues that need attention in classroom implementation from two aspects: implementation points and boundary conditions. The implementation of this pathway requires three points to be clarified.

First, the key to Stage 1 is "letting students establish the correspondence themselves" rather than "the teacher telling them the correspondence". When students match incorrectly, the pathway requires teachers not to correct them directly, but to guide them back to the object to observe again, so that the connection between direction and view is completed through self-correction. The role of the teacher is not to provide the correct answer, but to provide the conditions under which students discover the correct answer themselves.

Second, the key to Stage 2 is "facilitating the leap amid cognitive conflict" rather than "smooth transition". The transformation of students from reliance on objects to imaginal operation is inevitably accompanied by difficulties and disputes. When students assemble different solid figures from the same view, the teacher asks "how did you think about it", guiding students to externalize the reasoning process, compare and revise it in discussion, and complete the conversion from external operation to internal imagery amid moderate cognitive tension.

Third, the key to Stage 3 is "controlling the magnitude of variation" so as to guide students to identify the structure. The span between variants should be neither too large nor too small; when a large number of students encounter difficulties, the teacher needs to add intermediate transitional levels. After each level of variant is completed, students are guided to compare similarities and differences until they can summarize in their own words the general method of "first determining the reference direction → then extracting information from each direction one by one → finally restoring comprehensively".

This pathway is applicable to the new-lesson teaching of "Observing Objects" in the second learning stage of primary school (Grades 3–4); it requires group cooperation as the organizational form and small cubes as the operational learning tools. It should also be noted that, as a study of theoretical construction, the pathway proposed in this paper has not yet been systematically tested by large-sample teaching experiments, and its effect awaits verification by subsequent empirical research.

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